MONTE CARLO SIMULATION OF THE VOLUME AND NUMBER OF ROCKFALL FRAGMENTS USING TALUS DEPOSITS

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Volume of blocks produced by rockfall fragmentation can be regarded as a random variable having a site-dependent probability distribution. Once the probability model is identified, both the volume and the number of block can be simulated for a given rockfall volume using the inversion technique. Model identification requires a large number of blocks, which is not readily available in rockfall inventories. We used blocks of the talus slope, deposited by rockfalls along centuries, to overcome this shortcoming. The method was applied to a test site in the Andorra Principality. Volume of blocks of rockfall events was also available in the site and used to test the method. Block volume in the site follows a truncated power law which depends on the rockfall volume. 20000 distribution curves were simulated for several rockfall volumes. Both the inventoried and the simulated block volumes show a large variability even for a given rockfall volume. The inventoried volumes are scattered within the 90% band of the simulated ones. This finding shows that the simulation results are realistic.

Keywords: rockfalls, fragmentation, block volume, random simulation, validation.

INTRODUCTION

Fragmentation reduces the size of the falling rock pieces and increases the number of blocks of a propagating rockfall. As a consequence, rockfall volume-frequency curves are not directly applicable to the hazard assessment of fragmental rockfalls [1, 2]. The average hazard can be obtained by means of the estimation of the mean number of blocks of a given volume [1-4]. However, for an event based hazard analysis, the size and number of the blocks produced in events must be determined, as it was carried out in [5] using a fractal model. The random nature of the process of rockfall fragmentation has been clearly shown by in-situ tests [6, 7]. This latter suggests that Monte Carlo methods are useful for simulating blocks volumes and emulating fragmentation. This is the approach we used. The basis of the method and its application to a site in the Andorra Principality are described. The simulated block volume distributions are compared with those of events inventoried in the site.

GENERAL METHOD

The method assumes that the volume of blocks ($V_f$) resulting from rockfall events follow a given distribution, which depends on the site and on the rockfall volume. The first step is to infer this distribution. Using data from inventoried events would be the best way to do this. However, inventories usually contain only just the larger blocks. In sites where repeated rock-

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falls accumulate at the cliff bottom $V_f$ can be measured at talus deposits. Talus blocks should have a distribution similar to that of the rockfall events. The latter is true provided the conditions in the source and propagation areas did not change significantly in time. It is worth noting that $V_f$ has an upper limit, $V_{sup}$, which depends on the rockfall volume, $V_r$.

The next steps in the method are: a) fitting a probability model to the empirical distribution of the block volume, b) inferring the $V_{sup}$ value conditioned on $V_r$, and c) simulating block volumes, using the inverse equation of the fitted model, until $V_r$ is reached. Further details of the method are described below by means of their application to a talus of the Solà d’Andorra.

**SIMULATING BLOCK VOLUMES FOR THE SOLÀ D’ANDORRA**

**Distribution of volume of block fragments of inventoried rockfall events**

The Solà d’Andorra is a granodioritic cliff where rockfalls occur frequently menacing the Andorra la Vella city, the capital of the Principality. Thirteen rockfalls with a volume from 4 to 150 m$^3$ have been inventoried there since 1999 [2, 8]. All of them fragmented. On the other hand, the volume of 2041 blocks measured at the Borrasica – Forat Negre talus of the Solà d’Andorra was also available [8]. The minimum $V_f$ measured at the talus was 0.001 m$^3$.

Fig. 1 shows the empirical exceeding cumulative probability ($S = P(V_f \geq v)$) of $V_f$ for block-sized fragments of inventoried events and for talus blocks. Data corresponding to $V_f \geq 0.9$ m$^3$ for the event data set (48 blocks) and to $V_f \geq 0.2$ m$^3$ for the talus set (539 blocks) are plotted in Fig. 1. A clear rollover effect was found in below these values, which is regarded as due to undersampling of the smaller blocks. The empirical distributions of $V_f$ for talus blocks and for event blocks are quite similar. They can be well fitted by power laws that have exponents statistically identical. This is even clearer if the distribution of the talus blocks is calculated using only blocks larger than 0.9 m$^3$ (Fig. 1). Standard power laws can be fitted to these data (e.g. thin green line in Fig. 1); however it is worth noting that such a distribution has no upper limit ($V_{sup} = \infty$), which is not realistic and would lead to nonsense results in the simulation.

![Fig. 1 Empirical and fitted distributions of the volume of blocks measured for rockfall events and on the Borrasica – Forat Negre talus slope (Solà d’Andorra, Andorra Principality).](image)
The models fitted to the empirical data are truncated power laws, which general expression is:

$$S(V_f) = \frac{(V_{inf}/V_f)^\beta - (V_{inf}/V_{sup})^\beta}{1 - (V_{inf}/V_{sup})^\beta} \quad \text{(Eq. 1)}$$

Where $V_{inf}$ and $V_{sup}$ are the minimum and maximum limit values for $V_f$, respectively, and the exponent $\beta$ is a shape parameter. $S$ probability is null for $V_{sup}$, i.e. no blocks may be equal or larger than it. It must be highlighted that the estimate for $\beta$ exponent depends on $V_{sup}$. A volume greater than the largest block observed in the lower part of the talus, or even below in the fluvial plain, should be used for $V_{sup}$. The largest block found in the talus (46 m$^3$) is located in their middle part; however, it could be not representative of $V_{sup}$ because the lower part of the talus, where usually the largest blocks are located, is urbanised. The volume of the largest block identified in the cliff face (203 m$^3$) [8] was instead considered for $V_{sup}$. Using this latter bound, a value of 0.969 ± 0.022 was estimated for $\beta$, using the maximum likelihood method.

**Fig. 2.** Simulated $V_f$ distributions (20000) and empirical distributions of three recent events in the Solà d’Andorra for 10 m$^3$ rockfalls. Data are plot using linear scales (left) and log scales (right).

**Simulating the volume and number of fragments and comparison with events observed**

Block volumes were simulated using the inverse of Eq. 1, to show the block volume as depending on $S$, and giving random values to $S$ generated from the uniform distribution. To emulate fragments of a rockfall of a given volume $V_r$, random $V_f$ values were generated until their sum reached $V_r$, within an 1% tolerance margin. Sets of simulated $V_f$ within this margin were retained (otherwise were rejected). The number of simulated $V_f$ in a set is taken as corresponding to the number of blocks resulting from fragmentation of the rockfall.

Simulations for the test site were carried out setting $\beta$ to 0.980 and $V_{inf}$ to 0.2 m$^3$. It should be noted that $V_{sup}$ depends on $V_r$ in this case (i.e. it will be different than the used for the overall distribution of the talus blocks). $V_{sup}$ was determined, first, using the regression between the largest block ($V_{f, max}$) and $V_r$ in the inventoried rockfalls, and, second, carrying out several simulation trials for determining the equation relating the maximum $V_{f, max}$ (≈$V_{sup}$) and mean $V_{f, max}$ and then trying to fit the mean $V_{f, max}$ - $V_r$ relationship shown by the inventoried events.
Once $V_{sup}$ was calibrated, 20000 rockfall events were simulated for several $V_r$. Fig. 2 shows the results obtained for 10 m$^3$ rockfalls (cyan lines). The $V_r$ distributions for the events inventoried having this $V_r$ are also shown in the plots. To make easier the comparison between the simulated and the inventoried distributions, several lines showing the proportion of the simulated curves for each $V_r$ were also drawn (e.g., 95% of the simulated curves are located below the 95% line). The variability in the simulated cases is noticeable, as it can be expected for power law behaviour and for a very large number of runs, though it is also very large for the inventoried events. $V_r$ data from 11 rockfalls occurred in the Solà d’Andorra were used to check the method. They scattered between the 5 and 95% of the simulated cases and around the line corresponding to 50% of the simulations. This suggests that most of the curves simulated are realistic. Such results are encouraging though data from additional rockfall events are required in order to test the goodness of the method accurately.

CONCLUSIONS

The volume of blocks both in the Borrasica-Forat Negre talus site and in the inventoried rockfalls follows a truncated power law, which depends on the rockfall volume. Exponent $\beta$ ranges from 0.969 to 0.980 in the site depending on the value used for the upper limit of the distribution model. The volume of blocks in the inventoried events shows a large variability even for the same rockfall volume. The comparison of the simulated distributions and those of the events occurred in the site shows that the method provides realistic results. Major advantages of the method are its ability to reproduce the variability of the fragmentation process and to provide the distributions of the random variables related (e.g. the volume of the largest block for a given rockfall volume).

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